Op-Amp Circuit Analysis by Inspection

Eric Danson

August 19, 2024

1 General Method

1. To find the poles, set the input (v_{in}) to zero, or to find the zeros, set the output (v_{out}) to zero.
2. Find the equivalent resistance, R_{eq}, seen by each independent capacitor, C_{ind}, in the input and feedback networks.
   - Capacitors in series ($C_1 \parallel C_2$) or in parallel ($C_1 + C_2$) are combined to form a single independent capacitor.
3. Calculate the pole and zero frequencies as $\omega_{\text{x}} = 1/\left(R_{\text{eq}} C_{\text{ind}}\right)$.

2 Approximate Method

1. Series RC pairs in the input network form a pole, and pairs in the feedback network form a zero.
2. Parallel RC pairs in the input network form a zero, and pairs in the feedback network form a pole.
3. Calculate the pole and zero frequencies as $\omega_{\text{x}} = 1/(RC)$.

3 Examples

3.1 Lead Compensator

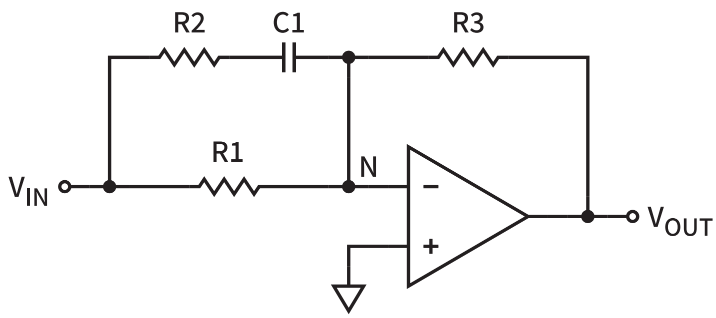


Figure 1: Lead compensator schematic.

The DC gain for the inverting configuration is

$$H_0 = -R_3/R_1 \tag{1}$$

by considering C_1 as an open circuit.

Obtain the poles by zeroing the input ($v_{\text{in}} = 0$) and finding the equivalent resistance seen by C_1. Note that node N, the op-amp's negative input terminal, is a virtual ground.

$$\omega_{\text{p}} = 1/(R_2 C_1) \tag{2}$$

Likewise, obtain the zeros by zeroing the output ($v_{\text{out}} = 0$).

$$\begin{aligned} \omega_{\text{z}} &= 1/[(R_1 + R_2)\, C_1] \\ &\approx 1/(R_1 C_1) \quad \text{for } R_1 \gg R_2 \end{aligned} \tag{3}$$

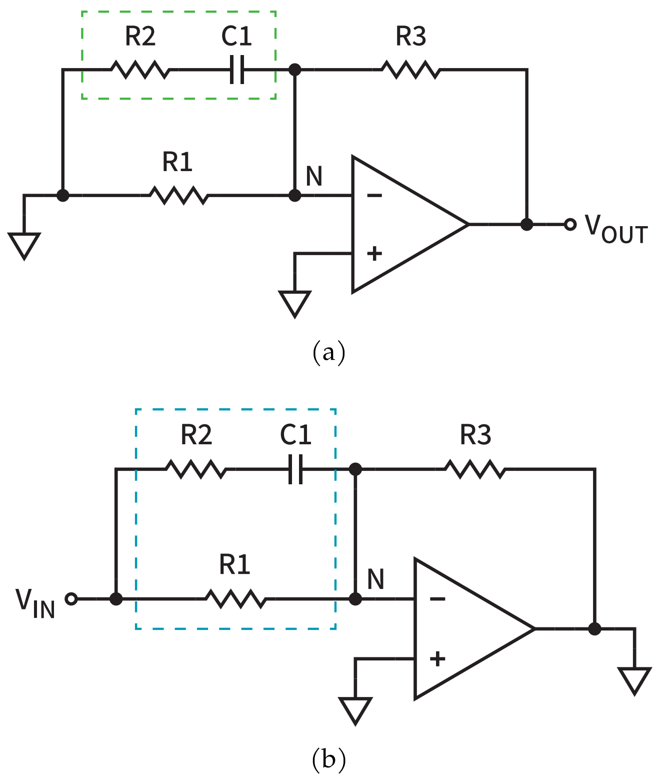


Figure 2: Lead compensator equivalent circuit for finding (a) poles and (b) zeros.

The complete transfer function can be put in the form

$$\begin{aligned} H(s) &= H_0 \frac{1 + s/\omega_{\mathrm{z}}}{1 + s/\omega_{\mathrm{p}}} \\ &= -\frac{R_3}{R_1} \cdot \frac{1 + s\left(R_1 + R_2\right) C_1}{1 + sR_2C_1} \\ &\approx -\frac{R_3}{R_1} \cdot \frac{1 + sR_1C_1}{1 + sR_2C_1} \quad \text{for } R_1 \gg R_2 \end{aligned} \tag{4}$$

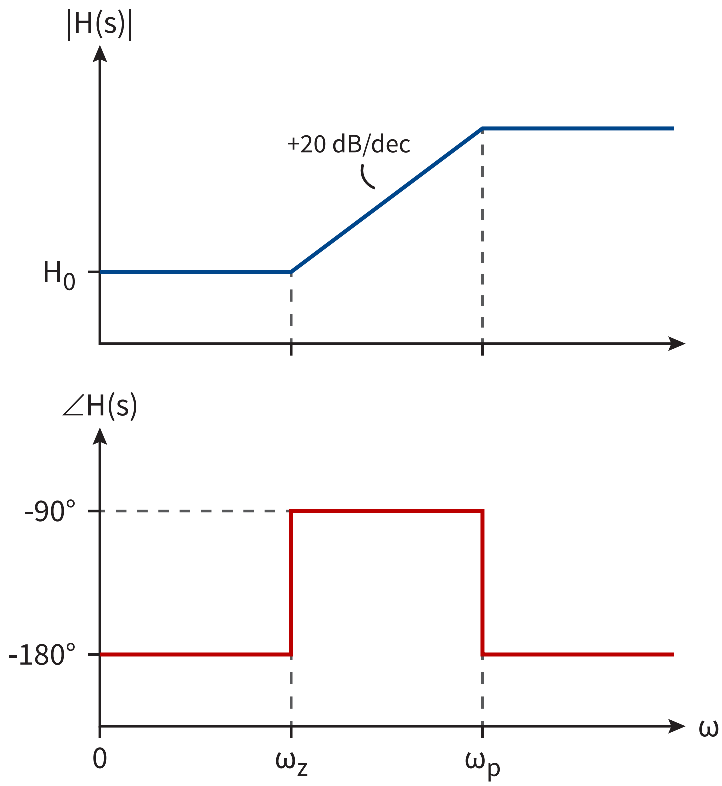


Figure 3: Lead compensator Bode plot.

3.2 Type 2 Compensator

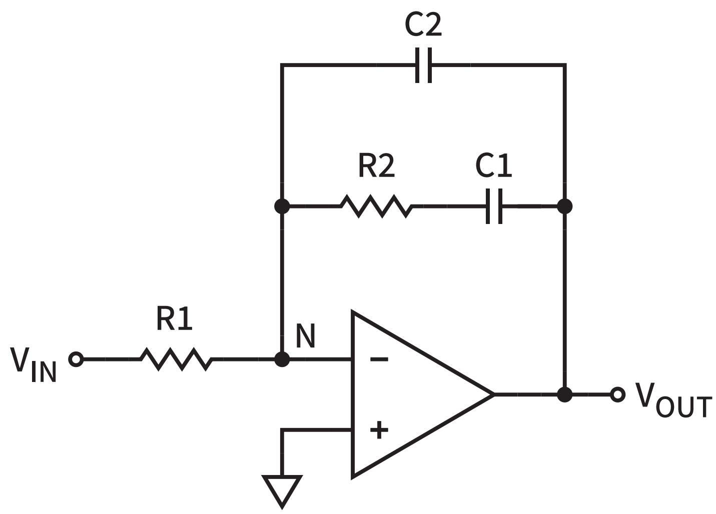


Figure 4: Type 2 compensator schematic.

The DC gain formula for an inverting configuration is not useful here because the feedback impedance at DC is undefined. Rather, the constant value in the integrator transfer function represents the gain at $\omega = 1\,\text{rad/s}$ and is determined by the input resistance and the feedback capacitance.

$$\begin{aligned} H_0 &= -1/[R_1\,(C_1 + C_2)] \\ &\approx -1/(R_1 C_1) \quad \text{for } C_1 \gg C_2 \end{aligned} \tag{5}$$

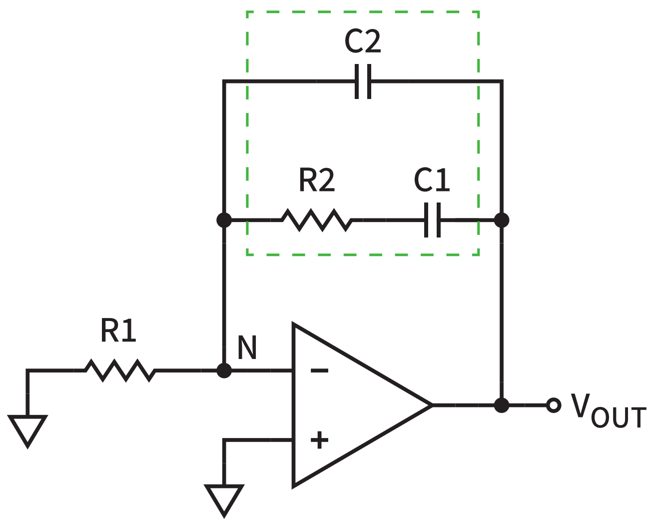
Obtain the poles by zeroing the input ($v_{\text{in}} = 0$) and finding the equivalent resistance seen by C_1 and C_2. Note that node N, the op-amp's negative input terminal, is a virtual ground.

$$\begin{aligned} \omega_{\text{p}} &= 1/[R_2\,(C_1 \parallel C_2)] \\ &\approx 1/(R_2 C_2) \quad \text{for } C_1 \gg C_2 \end{aligned} \tag{6}$$

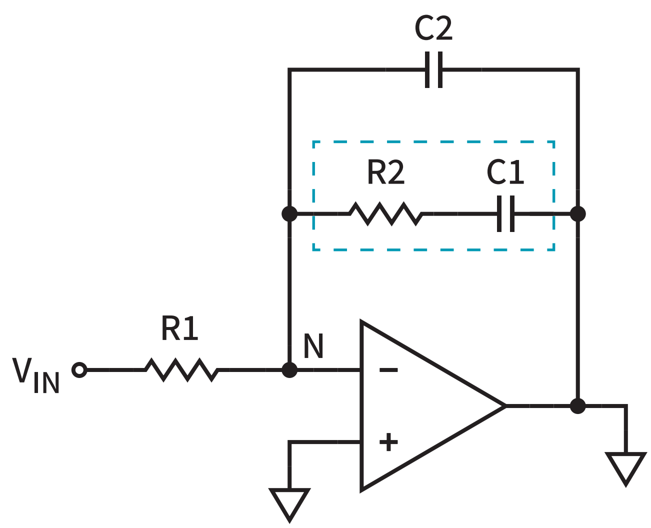
There is an additional $1/s$ term in the transfer function because C_1 and C_2 in the feedback network form an integrator.

Likewise, obtain the zeros by zeroing the output ($v_{\text{out}} = 0$).

$$\omega_{\text{z}} = 1/(R_2 C_1) \tag{7}$$



(a)



(b)

Figure 5: Type 2 compensator equivalent circuit for finding (a) poles and (b) zeros.

The complete transfer function can be put in the form

$$\begin{aligned} H(s) &= H_0 \frac{1 + s/\omega_z}{s\left(1 + s/\omega_p\right)} \\ &= -\frac{1}{R_1\left(C_1 + C_2\right)} \cdot \frac{1 + sR_2C_1}{s\left[1 + sR_2\left(C_1 \parallel C_2\right)\right]} \\ &\approx -\frac{1}{R_1C_1} \cdot \frac{1 + sR_2C_1}{s\left(1 + sR_2C_2\right)} \quad \text{for } C_1 \gg C_2 \end{aligned} \tag{8}$$

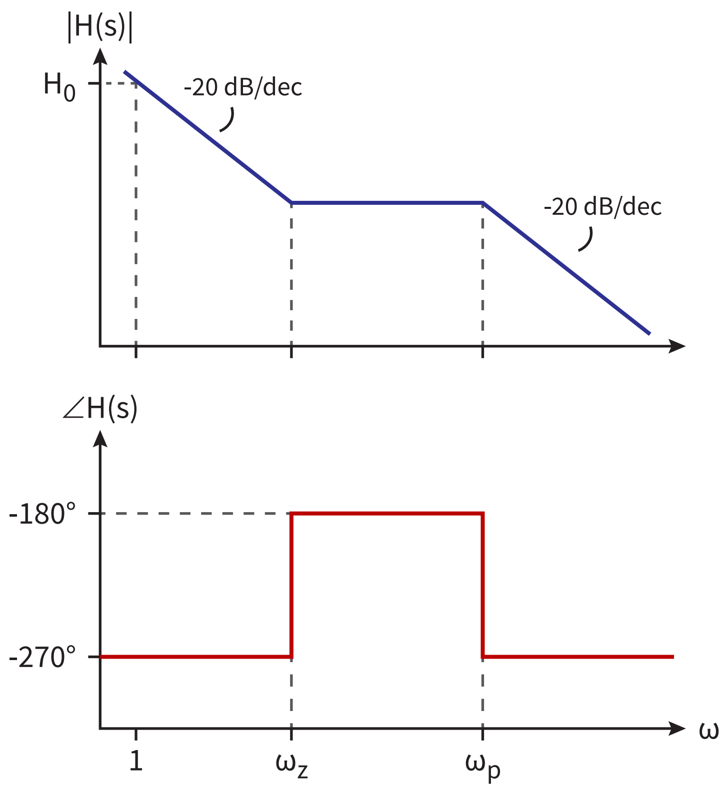


Figure 6: Type 2 compensator Bode plot.

3.3 Type 3 Compensator

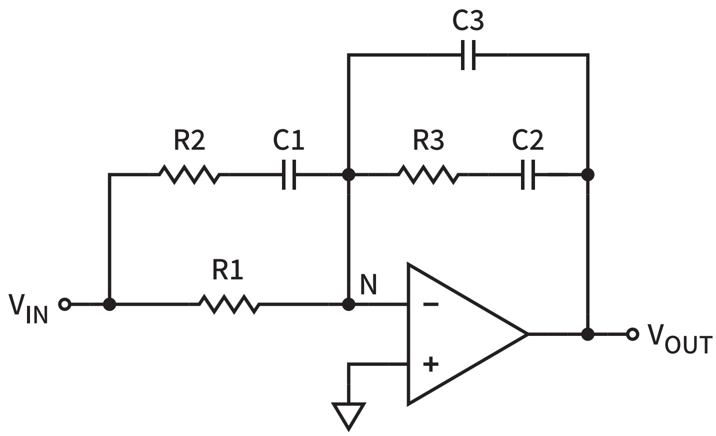


Figure 7: Type 3 compensator schematic.

The DC gain formula for an inverting configuration is not useful here because the feedback impedance at DC is undefined. Rather, the constant value in the integrator transfer function represents the gain at $\omega = 1\,\text{rad/s}$ and is determined by the input resistance and the feedback capacitance.

$$\begin{aligned} H_0 &= -1/[R_1\,(C_2 + C_3)] \\ &\approx -1/(R_1 C_2) \quad \text{for } C_2 \gg C_3 \end{aligned} \tag{9}$$

Obtain the poles by zeroing the input ($v_{\text{in}} = 0$) and finding the equivalent resistances seen by C_1, C_2, and C_3. Note that node N, the op-amp's negative input terminal, is a virtual ground.

$$\omega_{\text{p1}} = 1/(R_2 C_1) \tag{10}$$

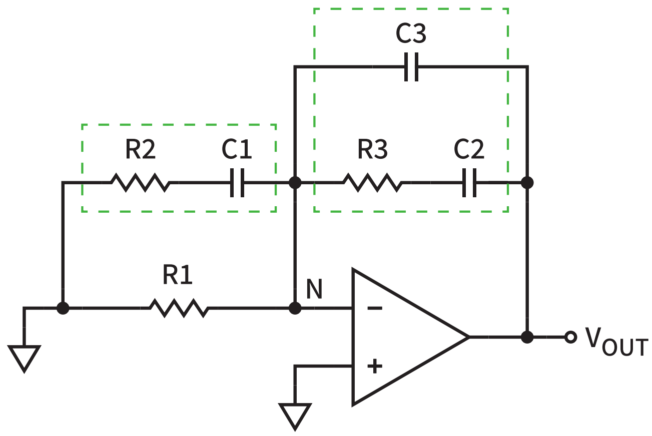
$$\begin{aligned} \omega_{\text{p2}} &= 1/[R_3\,(C_2 \parallel C_3)] \\ &\approx 1/(R_3 C_3) \quad \text{for } C_2 \gg C_3 \end{aligned} \tag{11}$$

There is an additional $1/s$ term in the transfer function because C_2 and C_3 in the feedback network form an integrator.

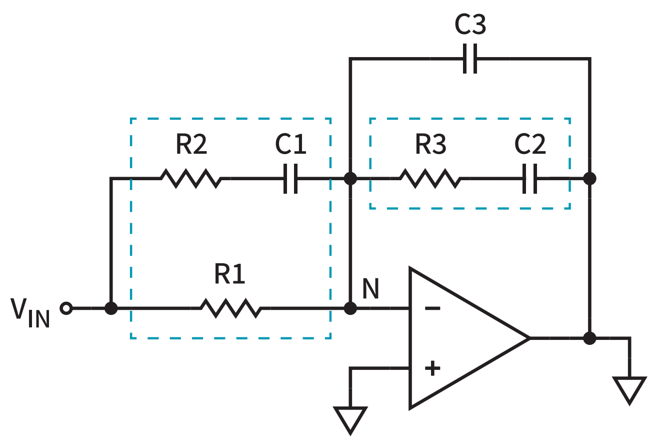
Likewise, obtain the zeros by zeroing the output ($v_{\text{out}} = 0$).

$$\begin{aligned} \omega_{\text{z1}} &= 1/[(R_1 + R_2)\,C_1] \\ &\approx 1/(R_1 C_1) \quad \text{for } R_1 \gg R_2 \end{aligned} \tag{12}$$

$$\omega_{z2} = 1/(R_3 C_2) \tag{13}$$



(a)



(b)

Figure 8: Type 3 compensator equivalent circuit for finding (a) poles and (b) zeros.

The complete transfer function can be put in the form

$$\begin{aligned}
H(s) &= H_0 \frac{(1 + s/\omega_{z1})(1 + s/\omega_{z2})}{s\left(1 + s/\omega_{p1}\right)\left(1 + s/\omega_{p2}\right)} \\
&= -\frac{1}{R_1(C_2 + C_3)} \cdot \frac{[1 + s(R_1 + R_2)C_1](1 + sR_3C_2)}{s(1 + sR_2C_1)[1 + sR_3(C_2 \parallel C_3)]} \\
&\approx -\frac{1}{R_1C_2} \cdot \frac{(1 + sR_1C_1)(1 + sR_3C_2)}{s(1 + sR_2C_1)(1 + sR_3C_3)} \quad \text{for } R_1 \gg R_2 \text{ and } C_2 \gg C_3
\end{aligned} \tag{14}$$

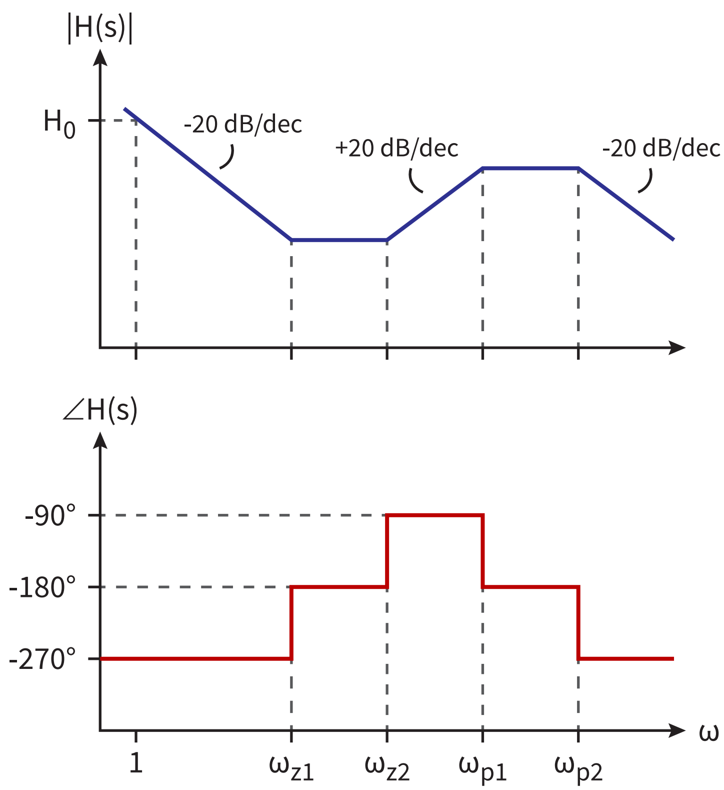


Figure 9: Type 3 compensator Bode plot.

4 Appendix

4.1 Lead Compensator Transfer Function Derivation

$$Z_{\mathrm{f}} = R_3 \tag{15}$$

$$\begin{aligned}
Z_{\mathrm{i}} &= R_1 \parallel \left(R_2 + \frac{1}{sC_1}\right) \\
&= \frac{R_1\left(R_2 + \frac{1}{sC_1}\right)}{R_1 + R_2 + \frac{1}{sC_1}} \\
&= \frac{R_1 + sR_1R_2C_1}{1 + s\left(R_1 + R_2\right)C_1}
\end{aligned} \tag{16}$$

$$\begin{aligned}
H(s) &= -\frac{Z_{\mathrm{f}}}{Z_{\mathrm{i}}} \\
&= -R_3 \cdot \frac{1 + s\left(R_1 + R_2\right)C_1}{R_1 + sR_1R_2C_1} \\
&= -\frac{R_3}{R_1} \cdot \frac{1 + s\left(R_1 + R_2\right)C_1}{1 + sR_2C_1} \\
&= H_0 \frac{1 + s/\omega_{\mathrm{z}}}{1 + s/\omega_{\mathrm{p}}}
\end{aligned} \tag{17}$$

$$H_0 = -\frac{R_3}{R_1} \tag{18}$$

$$\omega_{\mathrm{p}} = \frac{1}{R_2C_1} \tag{19}$$

$$\omega_{\mathrm{z}} = \frac{1}{\left(R_1 + R_2\right)C_1} \tag{20}$$

4.2 Type 2 Compensator Transfer Function Derivation

$$\begin{aligned} Z_\mathrm{f} &= \left(R_2 + \frac{1}{sC_1}\right) \parallel \frac{1}{sC_2} \\ &= \frac{\left(R_2 + \frac{1}{sC_1}\right)\frac{1}{sC_2}}{R_2 + \frac{1}{sC_1} + \frac{1}{sC_2}} \\ &= \frac{1 + sR_2C_1}{s\left(C_1 + C_2 + sR_2C_1C_2\right)} \end{aligned} \tag{21}$$

$$Z_\mathrm{i} = R_1 \tag{22}$$

$$\begin{aligned} H(s) &= -\frac{Z_\mathrm{f}}{Z_\mathrm{i}} \\ &= -\frac{1 + sR_2C_1}{s\left(C_1 + C_2 + sR_2C_1C_2\right)} \cdot \frac{1}{R_1} \\ &= -\frac{1}{R_1\left(C_1 + C_2\right)} \cdot \frac{1 + sR_2C_1}{s\left(1 + \frac{sR_2C_1C_2}{C_1+C_2}\right)} \\ &= -\frac{1}{R_1\left(C_1 + C_2\right)} \cdot \frac{1 + sR_2C_1}{s\left[1 + sR_2\left(C_1 \parallel C_2\right)\right]} \\ &= H_0 \frac{1 + s/\omega_\mathrm{z}}{s\left(1 + s/\omega_\mathrm{p}\right)} \end{aligned} \tag{23}$$

$$H_0 = -\frac{1}{R_1\left(C_1 + C_2\right)} \tag{24}$$

$$\omega_\mathrm{p} = \frac{1}{R_2\left(C_1 \parallel C_2\right)} \tag{25}$$

$$\omega_\mathrm{z} = \frac{1}{R_2C_1} \tag{26}$$

4.3 Type 3 Compensator Transfer Function Derivation

$$\begin{aligned}
Z_{\mathrm{f}} &= \left(R_3 + \frac{1}{sC_2}\right) \parallel \frac{1}{sC_3} \\
&= \frac{\left(R_3 + \frac{1}{sC_2}\right)\frac{1}{sC_3}}{R_3 + \frac{1}{sC_2} + \frac{1}{sC_3}} \\
&= \frac{1 + sR_3C_2}{s\left(C_2 + C_3 + sR_3C_2C_3\right)}
\end{aligned} \tag{27}$$

$$\begin{aligned}
Z_{\mathrm{i}} &= R_1 \parallel \left(R_2 + \frac{1}{sC_1}\right) \\
&= \frac{R_1\left(R_2 + \frac{1}{sC_1}\right)}{R_1 + R_2 + \frac{1}{sC_1}} \\
&= \frac{R_1 + sR_1R_2C_1}{1 + s\left(R_1 + R_2\right)C_1}
\end{aligned} \tag{28}$$

$$\begin{aligned}
H(s) &= -\frac{Z_{\mathrm{f}}}{Z_{\mathrm{i}}} \\
&= -\frac{1 + sR_3C_2}{s\left(C_2 + C_3 + sR_3C_2C_3\right)} \cdot \frac{1 + s\left(R_1 + R_2\right)C_1}{R_1 + sR_1R_2C_1} \\
&= -\frac{1}{R_1\left(C_2 + C_3\right)} \cdot \frac{\left(1 + sR_3C_2\right)\left[1 + s\left(R_1 + R_2\right)C_1\right]}{s\left(1 + \frac{R_3C_2C_3}{C_2+C_3}\right)\left(1 + sR_2C_1\right)} \\
&= -\frac{1}{R_1\left(C_2 + C_3\right)} \cdot \frac{\left[1 + s\left(R_1 + R_2\right)C_1\right]\left(1 + sR_3C_2\right)}{s\left(1 + sR_2C_1\right)\left[1 + sR_3\left(C_2 \parallel C_3\right)\right]} \\
&= H_0\frac{\left(1 + s/\omega_{\mathrm{z1}}\right)\left(1 + s/\omega_{\mathrm{z2}}\right)}{s\left(1 + s/\omega_{\mathrm{p1}}\right)\left(1 + s/\omega_{\mathrm{p2}}\right)}
\end{aligned} \tag{29}$$

$$H_0 = -\frac{1}{R_1\left(C_2 + C_3\right)} \tag{30}$$

$$\omega_{\mathrm{p1}} = \frac{1}{R_2 C_1} \tag{31}$$

$$\omega_{\mathrm{p2}} = \frac{1}{R_3\left(C_2 \parallel C_3\right)} \tag{32}$$

$$\omega_{\mathrm{z1}} = \frac{1}{\left(R_1 + R_2\right) C_1} \tag{33}$$

$$\omega_{\mathrm{z2}} = \frac{1}{R_3 C_2} \tag{34}$$